Equal Area method continued....

start with the swing equation.

$$M\frac{d^2\delta}{dt^2} = P_a \quad \text{, where, } M = J\omega$$

$$\frac{d^2\delta}{dt^2} = \frac{P_a}{M} = \frac{d\omega}{dt}$$

we would like to obtain an expression for the variation of the angular speed ω wrt P_a, we may write

$$d\omega = \frac{P_a}{M}\left(\frac{d\delta}{d\delta}\right)dt$$

but

$$\frac{d\delta}{dt} = \omega \quad \text{or} \quad d\delta = \omega\, dt$$

$$\therefore \quad d\omega = \frac{P_a}{M}\left(\frac{1}{\omega}\right)d\delta$$

$$\omega\, d\omega = \frac{P_a}{M}\, d\delta$$

Integrating

$$\int_{\omega_0}^{\omega} \omega\, d\omega = \frac{1}{M}\int_{\delta_0}^{\delta} P_a\, d\delta$$

we assume that $\omega_0 = 0$

$$\therefore \quad \frac{\omega^2}{2} = \frac{1}{M}\int_{\delta_0}^{\delta} P_a\, d\delta$$

$$\omega = \left[\frac{2}{M}\int_{\delta_0}^{\delta} P_a \, d\delta\right]^{1/2}$$

When the machine reaches the new steady state, δ is constant and the change in angular velocity is zero.

Then from the equation, δ starts at δ_0 and reaches δ_s, then we write:

$$\int_{\delta_0}^{\delta_s} P_a \, d\delta = \int_{\delta_0}^{\delta_s} (P_1 - P_e) \, d\delta = 0$$

or

$$\int_{\delta_0}^{\delta_1} (P_1 - P_e) \, d\delta - \int_{\delta_1}^{\delta_s} (P_1 - P_e) \, d\delta = 0$$

$$\boxed{= A_1 - A_2 = 0}$$

$$A_1 = P_1 (\delta_1 - \delta_0) - \int_{\delta_0}^{\delta_1} P_{max} \sin\delta \, d\delta$$

must be radians

$$A_2 = \int_{\delta_1}^{\delta_s} P_{max} \sin\delta \, d\delta - P_1 (\delta_s - \delta_1)$$

Evaluating integrals and substituting, we get:

$$\delta_s + \frac{P_{max}}{P_1}\cos\delta_s = \delta_0 + \frac{P_{max}}{P_1}\cos\delta_0$$

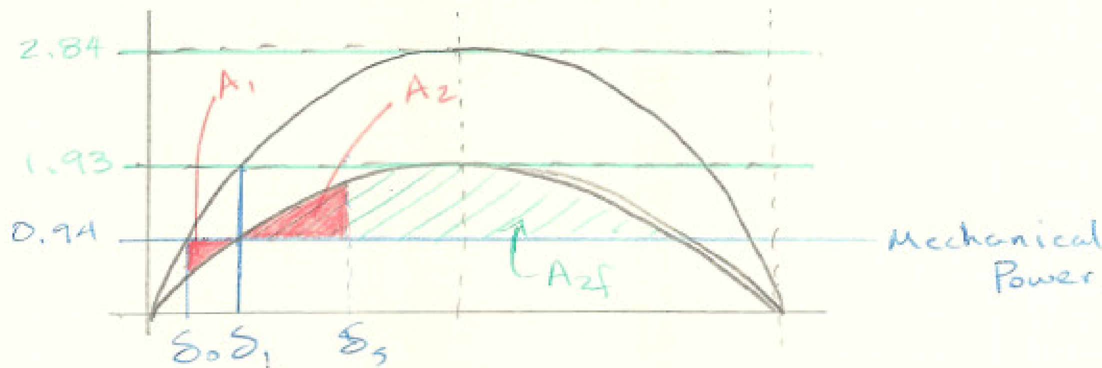
Trial and error will give a value for δ_s

Ex: Consider the system in the previous example, Determine whether the system is stable for a fault of an open circuit on line 2. If stable, determine δ_s.

SOL:

$$\delta_0 = 19.2^\circ * \frac{\pi}{180} =$$

$$\delta_1 = 29.15^\circ * \frac{\pi}{180} =$$



$$A_1 = 0.94(29.15 - 19.2)\frac{\pi}{180} - \int_{19.2}^{29.15} 1.93 \sin\delta \, d\delta$$

$A_1 = 0.0262$ units

$\delta_f = 180 - \delta_1 = 150.85^\circ$

$$A_{2f} = 1.93 \int_{\delta_1}^{\delta_f} \sin\delta \, d\delta - \left[(\delta_f - \delta_1)\frac{\pi}{180}\right] 0.94$$

$A_{2f} = 1.3745$

note: If $A_{2f} > A_1$, system is stable

to find δ_s, then...

$A_1 = A_2$

$$0.0262 = \int_{\delta_1}^{\delta_s} 1.93 \sin\delta (d\delta) - 0.94(\delta_s - \delta_1)\frac{\pi}{180}$$

$$0.0262 = 1.93(-\cos\delta_s + \cos 29.15^\circ) - 0.94(\delta_s - 29.15^\circ)\frac{\pi}{180}$$

here δ_s is in degrees, simplifying

$$1.93\cos\delta_s + 0.0164\,\delta_s = 2.1376$$

This is a nonlinear equation, by trial and error, we find that:

$$\delta_s = 39.39^\circ$$

note: In the above example, the equal area method was applied to one generator supplying an ∞ bus through 2 parrellel lines and for the loading indicated the system is stable.

The opening of one line may cause the generator to lose synchronism even though the load can be supplied over one single line

EX: Assume the system under consideration is devilering an active power of 1.8 p.u. with the same source voltage E as before. Determine whether the system will be stable after one line is removed.